Two-Time Scale Stabilization of Systems with Output Feedback

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The problem of constant-gain output feedback regulator design for linear systems with ill-conditioned dynamics is considered in the context of singular perturbation theory. A design approach is developed in which gains can be separately calculated to stabilize reduced-order slow and fast subsystem models. By employing the notion of combined control and observation spillover suppression, conditions are derived to assure that these gains will stabilize the full-order sytem, assuming sufficient frequency separation between the slow and fast subsystems. An optimal control design procedure is described in which the spillover suppression conditions are satisfied by adjoining penalty functions to the subsystem performance indices. The theory is demonstated in a controller design for a flexible space structure.

Introduction

CONSTANT-GAIN output feedback controllers for linear multivariable systems have an important advantage over those based on full-state feedback in that they are simple to implement. Since the system output is typically of lower dimension than the system state, restricting the control structure to a linear combination of outputs avoids the need to include a state observer in the controller. However, this complicates the task of determining linear-quadratic (LQ) optimal gains. The necessary conditions for the optimal output feedback problem result in a system of coupled nonlinear matrix equations.[1] If the design model contains slow and fast modes, the difficulties in solving these equations are compounded by the presence of numerical ill-conditioning.

When dealing with large-scale systems, such as modern flexible spacecraft, the high dimensionality and ill-conditioning of the state matrix motivate the use of model order reduction schemes for making the control design problem tractable. In this paper, singular perturbation theory (SPT) is employed to decompose the dynamics into separate numerically well-conditioned subsystems, each of lower order than the original plant. It has been shown[2] that, in the case of full-state feedback, application of SPT to the LQ regulator problem directly separates the controller design into slow and fast subproblems. For output feedback, this occurs only for a restricted class of output structures.[3,4] It will be seen that, in the general case of designing output feedback gains for a system decomposed via SPT, a single set of gains must stabilize both subsystems simultaneously.

In systems with many sensors and actuators, the slow and fast subsystems created through an SPT decomposition exhibit rank deficiency (or near deficiency) in the subsystem input/output structures. Reference 5 describes a two-time scale output feedback design procedure that exploits input rank deficiency to separate slow and fast subsystem stabilization into two separate tasks. In this paper, combined control and observation spillover suppression exploits rank deficiency in both the input and output structures. This broadens the set of admissible gains for which the slow and fast subsystem designs can be separated. For brevity, as well as for reasons that will become transparent shortly, we will refer to this approach as gain spillover suppression (GAS). A numerical example will illustrate the design procedure.

Problem Statement

Consider the system

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1 u \qquad x_1 \in R^{n_1} \tag{1}$$

$$\epsilon\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2 u \qquad x_2 \in R^{n_2} \tag{2}$$

with output

$$y = C_1 x_1 + C_2 x_2 \qquad y \in R^p \tag{3}$$

where $0 < \epsilon \ll 1$ and A_{22} is assumed invertible. The control takes the form

$$u = -Gy \qquad u \in R^m \tag{4}$$

Because of the slow and fast dynamics in Eqs. (1) and (2), a simplified plant model is obtained by setting $\epsilon = 0$ in Eq. (2), resulting in the low frequency model,

$$\dot{x} = A_0 x + B_0 u \qquad x \in R^{n_1} \tag{5}$$

$$y = C_0 x + D_0 u \tag{6}$$

$$A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21} \qquad B_0 = B_1 - A_{12}A_{22}^{-1}B_2$$

$$C_0 = C_1 - C_2 A_{22}^{-1}A_{21} \qquad D_0 = -C_2 A_{22}^{-1}B_2 \tag{7}$$

Note the presence of control feedthrough in Eq. (6). Substituting Eq. (6) into Eq. (4) results in

$$u = -G^{\circ} C_0 x \tag{8}$$

$$G^{\circ} = (I + GD_0)^{-1} G \tag{9}$$

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so that Eq. (8) is the equivalent feedback control for the reduced system. Because of the pesence of the inverse in Eq. (9), we consider only G, such that

$$\rho(I+GD_0)=m \tag{10}$$

It is shown in Ref. 6 that the class of gains satisfying Eq. (10) includes all of the bounded G that stabilize the system of Eqs. (1-4). In order to calculate the feedback gain matrix for Eqs. (1-4) based on the low-frequency model of Eqs. (5) and (8), one designs G° and then inverts Eq. (9) to obtain

$$G=G^\circ(I-D_0G^\circ)^{-1} \tag{11}$$

for implementation. The inverse in Eq. (11) exists if Eq. (10) holds, in which case G and G° are locally one-to-one.[6] This assures that the design problem is well defined.

The following lemma, proved in Ref. 7, describes an important eigenvalue property that can be used to relate the closed-loop stability of the full- and reduced-order sytems.

Lemma 1: Consider the system

$$\dot{v}_1=\Gamma_1 v_1+\Gamma_2 v_2 \tag{12}$$

$$\epsilon\dot{v}_2=\Gamma_3 v_1+\Gamma_4 v_2 \tag{13}$$

where Γ_4 is nonsingular. As $\epsilon\to 0$, the spectrum of Eqs. (12) and (13) is given by

$$\sigma=\sigma[\Gamma_0+\mathcal{O}(\epsilon)]\cup\sigma[\Gamma_4+\mathcal{O}(\epsilon)]/\epsilon \tag{14}$$

$$\Gamma_0=\Gamma_1-\Gamma_2\Gamma_4^{-1}\Gamma_3 \tag{15}$$

It has been shown[8] that, for the closed-loop system of Eqs. (1-4),

$$\Gamma_0=A_0-B_0G^\circ C_0 \tag{16}$$

$$\Gamma_4=A_{22}-B_2GC_2 \tag{17}$$

so that a direct consequence of this lemma is that, if G satisfies

$$(A_0-B_0G_0C_0) \quad \text{is Hurwitz} \tag{18}$$

$$(A_{22}-B_2GC_2) \quad \text{is Hurwitz} \tag{19}$$

subject to Eq. (9), then G will stabilize the full-order system of Eqs. (1-4) for sufficiently small ϵ. The control designer's task, then, is to design G based on Eqs. (16), (17), and (9).

Stabilization with Gain Spillover Suppression

In this section, conditions are given for using GSS to separate the design of G into a two-step process. One gain matrix, G_1, is designed so as to satisfy Eq. (18) without disturbing the eigenstructure of Eq. (17). The other, G_2, is designed to satisfy Eq. (19) without affecting Eq. (16). The implemented gain takes the form

$$G=G_1+G_2 \tag{20}$$

It will be shown that this particular ordering of the design steps is necessary and that it does not impose any additional restriction on the implemented gain.

Suppose that A_{22} is "sufficiently" stable. In this case, let $G_2=0$, so that $G=G_1$ in Eq. (20). In order to avoid gain spillover into the fast dynamics, we require

$$B_2G_1C_2=0 \tag{21}$$

The following lemma provides an easily enforced constraint for satisfying Eq. (21). This lemma and lemmas 3 and 4 are proved) in the Appendix.

Lemma 2: Equation (21) holds if

$$B_2G_1^\circ C_2=0 \tag{22}$$

where

$$G_1^\circ=(I+G_1D_0)^{-1}G_1 \tag{23}$$

Moreover, if Eq. (22) holds, then the inverse in Eq. (23) exists and is given by

$$(I+G_1D_0)^{-1}=(I-G_1D_0) \tag{24}$$

The slow subsystem design thus consists of satisfying Eqs. (18) and (22), where $G^\circ=G_1^\circ$ in Eq. (18). Once a satisfactory G_1° is obtained, G_1 is calculated using

$$G_1=G_1^\circ(I-D_0G_1^\circ)^{-1}=G_1^\circ(I+D_0G_1^\circ) \tag{25}$$

The second of Eq. (25) can easily be verified from Eq. (22) and the form of D_0 in Eq. (7). Also, note from Eq. (24) and the form of D_0 that, if one exploits only the rank deficiency in the fast subsystem input matrix—that is, if one insists that

$$B_2G_1^\circ=0 \tag{26}$$

then $G_1^\circ=G_1$, so that G_1 may be directly designed to stabilize $A_0-B_0G_1C_0$ subject to the control spillover constraint of Eq. (26).

Now, suppose that the fast dynamics require improvement. In this case, G_2 is designed to stabilize the fast dynamics without spilling over into the slow dynamics. The design criteria are

$$(A_{22}-B_2G_2C_2) \text{ is Hurwitz} \tag{27}$$

$$B_0[I+(G_1+G_2)D_0]^{-1}(G_1+G_2)C_0=B_0G_1^\circ C_0 \tag{28}$$

where the spillover condition [Eq. (28)] is obtained from Eqs. (9), (18), and (20). One immediately notes that, if $D_0=0$, Eq. (28) reduces to a form that mirrors the slow subsystem GSS condition,

$$B_0G_2C_0=0 \tag{29}$$

The following Lemma provides a necessary and sufficient condition on G_2 for satisfaction of Eq. (28).

Lemma 3: Equation (28) holds if

$$B_0(I-G_1D_0)G_2(I-D_0G_1)C_0=0 \tag{30}$$

Despite the fact that Eq. (30) is dependent on G_1, the slow subsystem gain has no effect on the fast subsystem dynamics.

Lemma 4: Given that G_1 satisfies Eq. (21) and G_2 satisfies Eq. (30),, $B_2G_2C_2$ is not a function of G_1.

Lemma 4 implies that no flexibility is lost by adopting a two-step design procedure in which G_1 is treated as a constant during the design of G_2 satisfying Eq. (30). Furthermore, it is shown in the proof of Lemma 4 that we have the following decomposition

$$G_2=N_{B_0}+N_{C_0} \tag{31}$$

where

$$B_0N_{B_0}=N_{C_0}C_0=0 \tag{32}$$

Thus, the freedom in selecting G_2 depends only on the rank deficiency in the reduced system input and output matrices.

On the basis of the preceding development, the following theorem is stated:

Theorem: Let G_1° be an asymptotically stabilizing feedback gain for the reduced system $\{A_0, B_0, C_0\}$ that satisfies the GSS constraint of Eq. (22). Let G_2 be an asymptotically stabilizing feedback gain for the fast subsystem $\{A_{22}, B_2, C_2\}$ satisfying the GSS constraint of Eq. (30). Then

$$G = G_1^\circ (I + D_0 G_1^\circ) + G_2 \tag{33}$$

stabilizes the full-order system of Eqs. (1-4) for sufficiently small ϵ.

Moreover, the closed-loop spectrum will be

$$\lambda_i[(A_0 - B_0 G_1^\circ C_0) + \mathcal{O}(\epsilon)] \qquad i = 1, \ldots, n_1$$

$$\lambda_i[(A_{22} - B_2 G_2 C_2) + \mathcal{O}(\epsilon)]/\epsilon \qquad i = n_1 + 1, \ldots, n$$

An easily implemented approach to enforcing the GSS conditions [Eqs. (22) and (30)] is developed in the next section.

LQ Design Procedure

In this section, LQ optimal control theory is applied to the problem of determining the G that stabilizes the matrix

$$\hat{A} = A - BGC \tag{34}$$

subject to the constraint

$$MGP = 0 \tag{35}$$

which corresponds to the general form of the conditions stated in the theorem. Under the assumption that a solution exists, this is done by defining the performance index

$$J_0 = E_{x_0}\left\{\int_0^\infty x^T Q x + u^T R u \mathrm{d}t\right\} + \nu \|MGP\|^2 \tag{36}$$

for the dynamics

$$\dot{x} = Ax + Bu \qquad x(0) = x_0 \tag{37}$$

$$u = -GCx \tag{38}$$

In Eq. (36), $Q = \Gamma^T \Gamma$ such that $\{\Gamma, A\}$ is detectable and $R > 0$. The notation $E_{x_0}\{\cdot\}$ denotes expectation with respect to the random initial state x_0 where, for simplicity, it is assumed that x_0 is uniformly distributed on the unit sphere, so that $E\{x_0 x_0^T\} = I$. The notation $\|\cdot\|$ denotes the inner matrix norm

$$\|w\|^2 = \mathrm{tr}\{w^T w) \tag{39}$$

and $\nu \geq$ is chosen sufficiently large that $\|MGP\| \to 0$.

Following Ref. 9, the Lagrangian is written,

$$\pounds(G,K,L) = \mathrm{tr}\{K\} + \mathrm{tr}\{S(G,K)L^T\} + \nu\|MGP\|^2 \tag{40}$$

$$S(G,K) = \hat{A}^T K + K\hat{A} + Q + C^T G^T R G C = 0 \tag{41}$$

The first-order necessary conditions for optimality are

$$\partial\pounds/\partial G\Big|_* = 0 \quad \partial\pounds/\partial K\Big|_* = 0 \quad \partial\pounds/\partial L\Big|_* = 0 \tag{42}$$

where the * indicate that the gradients are evaluated at the optimal values of G, K and L. The * notation is henceforth suppressed, since the gradients are assumed evaluated at their optimal values unless specified otherwise. Expanding the gradients in Eq. (42), we have

$$-B^T K L C^T + RGCLC^T + \nu M^T M G P P^T = 0 \tag{43}$$

$$\hat{A}L + L\hat{A}^T + I = 0 \tag{44}$$

$$S(G,K) = 0 \tag{45}$$

These necessary conditions can be solved by using the algorithm described below. This algorithm satisfies the sufficient conditions for numerical convergence given in Ref. 10, is simple to implement, and has demonstrated a "fast" rate of convergence in practice.

0) Choose any G rendering $\hat{A}$ Hurwitz. Set $i = 0$ and $G = GCC^+$.

1) Solve Eqs. (42) and (43) for L^i, K^i.

2) On the basis of Eq. (39), evaluate

$$\Delta G^i = R^{-1}[B^T K^i L^i C^T - \nu M^T M G^i P P^T](CL^i C^T)^+ - G^i \tag{46}$$

3) Set

$$G_1^{i+1} = G^i + \alpha \Delta G^i \tag{47}$$

where $\alpha \epsilon (0,1]$ is chosen to ensure that

$$J_{i+1} < J_i = \mathrm{tr}\{K^i\} + \nu\|MG^i P\| \tag{48}$$

4) Set $i = i + 1$ and go to step 1.

Notes:

a) In Eq. (46) and step 0, $(\cdot)^+$ denotes the pseudoinverse. In the case where $\rho(C) = p$, $(CLC^T)^+ = (CLC^T)^{-1}$; however, this is not generally the case. Using the characteristics of the pseudoinverse, it can be shown that the columns of the transposed incremental gain $(\Delta G^i)^T$ will always lie wholly in $im\{C^T\}$. Because of this fact, the algorithm satisfies the requirements in Ref. 10 for guaranteeing convergence.

b) A simple means of supplying an initial stabilizing gain for step 0 is given in Ref. 11.

c) Since the design is carried out separately for the reduced and fast subsystem models, convergence is not adversely affected by small values of ϵ.

d) The potentially destabilizing effect of gain spillover can be reduced to any desired degree by choosing ν sufficiently large, so long as the conditions of the theorem are met.

Numerical Example

Figure 1 illustrates a large flexible space structure used to test the LQ design procedure. Data for this system came from

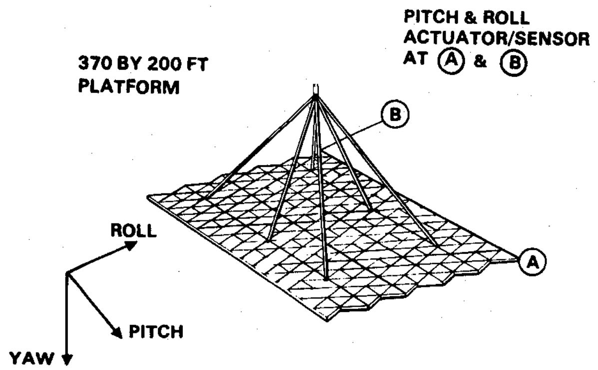

Fig. 1 Flexible space structure.

Ref. 12. For design purposes, a four-mode model was used,

$$A_{11}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -(.42)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -(0.42)^2 & 0 \end{bmatrix}, \quad A_{12}=0$$

$$A_{22}/\epsilon=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -(2.1)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -(2.2)^2 & 0 \end{bmatrix}, \quad A_{21}=0$$

$$\begin{bmatrix} B_1 \\ B_2/\epsilon \end{bmatrix}=\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ -0.92 & -1.4 & 0.92 & -1.4 \\ 0 & 0 & 0 & 0 \\ 0.65 & 1.6 & 0.65 & -1.6 \\ \hline 0 & 0 & 0 & 0 \\ 1.4 & -1.0 & 1.4 & 1.0 \\ 0 & 0 & 0 & 0 \\ 2.05 & -0.80 & -2.0 & -0.80 \end{array}\right]$$

$$C_1=\begin{bmatrix} 0 & -1.8 & 0 & 1.3 \\ 0 & -2.7 & 0 & 3.2 \\ 0 & 1.8 & 0 & 1.3 \\ 0 & -2.7 & 0 & -3.2 \end{bmatrix}$$

$$C_2=\begin{bmatrix} 0 & 2.9 & 0 & 4.1 \\ 0 & -2.1 & 0 & -1.6 \\ 0 & 2.9 & 0 & -4.1 \\ 0 & 2.1 & 0 & -1.6 \end{bmatrix}$$

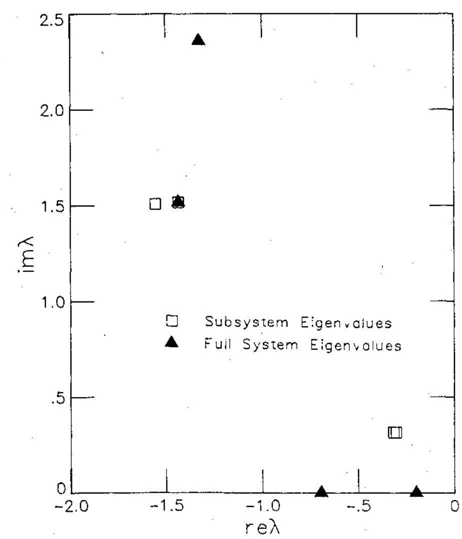

Fig. 2 Closed-loop eigenvalues without spillover suppression.

In this system, the third and fourth modes are approximately five times faster than the first and second modes. When describing the system in the form of Eqs. (1) and (2), one would naturally choose $\epsilon = 1/5$ and calculate A_{22} and B_2 accordingly. It should be noted, however, that the solution is independent of ϵ. In the reduced-order model $D_0 = 0$ and, in the absence of control feedthrough, the GSS constraint for the fast subsystem reduces to the form of Eq. (29).

The penalty weights employed in the slow and fast subsystem designs were:

$$Q_0 = \text{diag}[0,0.065,0,0.065], \quad R_0 = I$$

$$Q_2 = \text{diag}[0,1.3,0,1.0], \quad R_2 = I$$

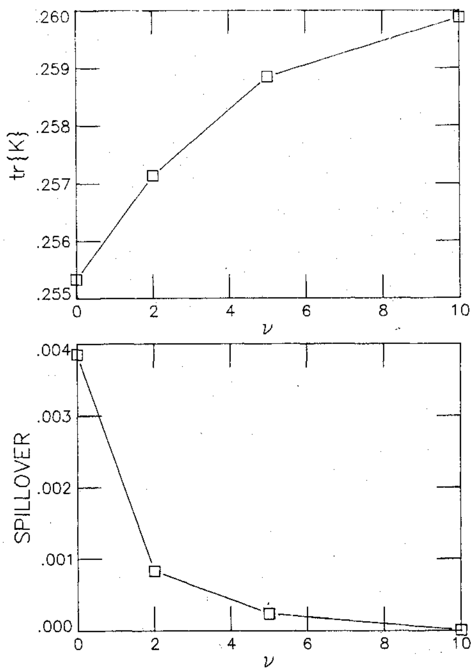

Fig. 3 Slow subsystem design.

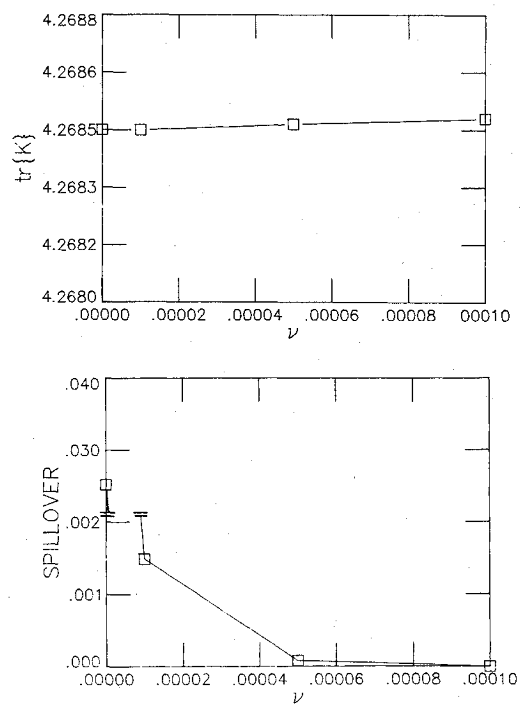

Fig. 4 Fast subsystem design.

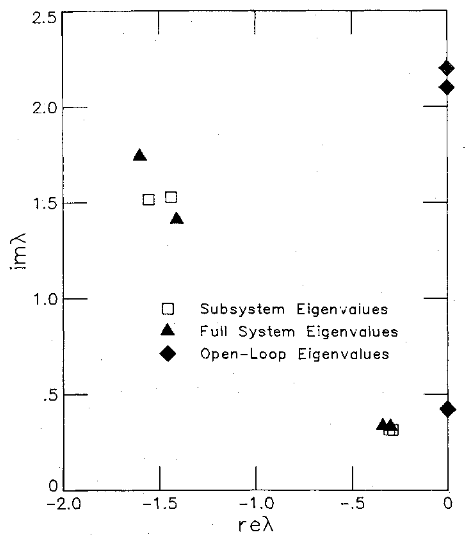

Fig. 5 Closed-loop eigenvalues with spillover suppression.

Figure 2 shows the upper half-plane closed-loop eigenvalues due to G formed from optimal subsystem designs without GSS ($\nu_0 = \nu_2 = 0$). The intended closed-loop eigenvalues for slow and fast subsystems are also shown. Note that the gain spillover distorts the response of the full-order system, tending to destabilize two of the modes. Figures 3 and 4 show the variation of integral quadratic performance and spillover penalty for the slow and fast subsystems, respectively. Figure 5 shows the upper half-plane closed-loop eigenvalues for the gain resulting from choosing $\nu_0 = 10$ and $\nu_2 = 0.001$ for design values. The degradation in integral quadratic cost due to the constraint of GSS for the slow subsystem was less than 1.8%. The degradation in the fast subsystem was negligible. The difference between the subsystem eigenvalues and the full-system eigenvalues is a consequence of the two-time-scale design based on reduced-order models. In general, this will depend on the relative speeds of the open-loop subsystems, for which the parameter ϵ simply acts as a measure.

Conclusion

Although the two-time scale constant-gain output feedback problem does not naturally decompose into separate slow and fast subproblems, it has been shown that such a separation can be made if the subsystems possess rank deficiency or near-rank deficiency in their input/output structures. This was done by exploiting the notion of two-way control and observation spillover suppression. Necessary and sufficient conditions were stated for enforcing these constraints and a linear-quadratic design procedure was described. The theory was applied in a numerical example.

Appendix

Proof of Lemma 2

Given that

$$B_2 G_1 C_2 = 0 \tag{A1}$$

and recalling that

$$D_0 = -C_2 A_{22}^{-1} B_2 \tag{A2}$$

one easily verifies that

$$(I + G_1 D_0)^{-1} = (I - G_1 D_0) \tag{A3}$$

Now, using Eqs. (23), (A1), and (A3),

$$B_2 G_1^\circ C_2 = B_2 (I - G_1 D_0) G_1 C_2 = 0 \tag{A4}$$

The converse is shown by using Eq. (25),

$$B_2 G_1 C_2 = B_2 G_1^\circ (I + D_0 G_1^\circ) C2 \tag{A5}$$

By Eq. (22), the right side of Eq. (A5) is zero, completing the proof.

Proof of Lemma 3

It was shown in Ref. 5 that, given that A_{22} is invertible, then $(I + GD_0)$ is invertible for all G_2 stabilizing the fast dynamics. Employing a standard matrix identity,

$$(I + D_0 G_2)^{-1} = I - D_0 (I + G_2 D_0)^{-1} G_2 \tag{A6}$$

so that $(I + D_0 G_2)^{-1}$ exists and applying the same identity to the inverse in the left side of Eq. (28), along with Eqs. (A1-A3) leads to

$$[I + (G_1 + G_2) D_0]^{-1} = (I - G_1 D_0)[I - G_2 (I + D_0 G_2)^{-1} D_0] \tag{A7}$$

Thus, the left side of Eq. (28) can be expressed as

$$B_0 [I + (G_1 + G_2) D_0]^{-1} (G_1 + G_2) C_0 = B_0 (I - G_1 D_0)$$
$$[I - G_2 (I + D_0 G_2)^{-1} D_0](G_1 + G_2) C_0 \tag{A8}$$

so that Eq. (28) becomes

$$B_0 (I - G_1 D_0) G_2 [I - (I + D_0 G_2)^{-1} D_0 (G_1 + G_2)] C_0 = 0 \tag{A9}$$

After some algebra, Eq. (A9) yields

$$B_0 (I - G_1 D_0) G_2 (I + D_0 G_2)^{-1} (I - D_0 G_1) C_0 = 0 \tag{A10}$$

It is easy to verify that

$$G_2 (I + D_0 G_2)^{-1} = (I + G_2 D_0)^{-1} G_2 \tag{A11}$$

This implies that

$$\ker\{G_2 (I + D_0 G_2)^{-1}\} = \ker\{G_2\} \tag{A12}$$

which implies that Eq. (A10) holds if Eq. (3) is true.

Proof of Lemma 4

A matrix N satisfying

$$MNP = 0 \tag{A13}$$

can be written

$$N = N_m + N_p \tag{A14}$$

where

$$MN_m = 0 \text{ and } N_p P = 0 \tag{A15}$$

In Eq. (30),

$$M = B_0 (I - G_1 D_0) \tag{A16}$$

$$P = (I - D_0 G_1) C_0 \tag{A17}$$

so that

$$G_2 - (I + G_1 D_0) N_{B_0} + N_{C_0} (I + D_0 G_1) \tag{A18}$$

where $(I + D_0 G_1) = (I - D_0 G_1)^{-1}$. Because of Eqs. (A1) and (A2), when G_2 from Eq. (A18) is substituted into $B_2 G_2 C_2$, G_1 cancels so that

$$B_2 G_2 C_2 = B_2 (N_{B_0} + N_{C_0}) C_2 \tag{A19}$$

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